



# **CORPORATE ACTIONS PRICING MANUAL**

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## INTRODUCTION

This manual presents the calculation methodologies for corporate events emphasizing on the ones considered complex, such as Subscription Bonus Issues on warrants or debentures. A corporate event is addressed in two stages. The first is on the ex-date when the stock is traded ex of the corporate event. The ex-price and the subscription right value are estimated simultaneously in order to provide the reference values to open the trading session, to adjust listed option exercise prices, for flexible option barriers, and for adjusting the participation of index component shares. The second is on the financial settlement date of the subscription right and involves the borrower of shares in a rental position to the lender. In this second calculation, only the right price is estimated, and the share price is already known (closing price). There is also other events covered by this manual such as voluntary events with subscription bonus.

## NOTATION

- $P_{com}$  : last price entitled to benefits
- $P_{ex}$  : ex-benefit.
- $w$ : subscription percentage.
- $T$ : subscription exercise expiration date.
- $\tilde{T}$ : last date to exercise the subscription bonus.
- $S(t)$ : share price on date  $t$ .
- $K$ : subscription price.
- $\tilde{K}_b$  : subscription bonus price.
- $\tilde{K}$  : subscription bonus exercise price.  $P(t)$ : value on date  $t$  of the subscription asset.
- $q_b$ : bonus amount per subscribed share.
- $q_a$ : share amount per subscription bonus.
- $1_b$ : indicator function, gives 1 when  $b$  is true and 0 otherwise.
- $RV$ : subscription right value.
- $\sigma_T$ : share price volatility for the time frame  $T$ .
- Call ( $\cdot$ ): value of a call option determined by the Black-Scholes model (see section A.5).
- $E[\cdot]$ : expected value of a random variable.

## 1 CALCULATING ORDINARY CORPORATE ACTION

The idea of pricing the ex-benefit stock price is to maintain the shareholder's portfolio financial value from the closing day with rights to the event to the open day without rights.

### 1.1 Cash actions

Contemplates actions like dividends, other cash distributions, capital refunds, interest payments, income, and amortizations.

$$P_{com} = P_{ex} + X \quad (1.1)$$

The solution of the equation (1.1) returns the value of  $P_{ex}$ , in other words,  $P_{ex} = P_{com} - X$ .

### 1.2 Quantity assets actions

Contemplates actions like stock split, stock merge and bonus share. To bonus with percentual  $B$ , the solution of the equation (1.2) returns the value of  $P_{ex}$ . To stock split and stock merge, the solution comes from equation (1.3), where  $Q$  is the proportion of the split or merge.

$$P_{ex} = \frac{P_{com}}{(1+B)} \quad (1.2)$$

$$P_{ex} = \frac{P_{com}}{Q} \quad (1.3)$$

### 1.3 BDR spin-off actions

The idea of pricing the ex-benefit price is to use available trading information in the market of the underlying asset. To calculate the price on the date of transition from with benefit to ex-benefit the following steps are applied, according to available information. Where  $TC$  is the exchange rate of Brazilian Real per currency which the underlying asset is traded,  $F$  is the parity factor and  $q$  is the proportion of the new company shares received per old company share.

- i. If, after a stock split, the underlying asset of the new company has a trading price  $P_{wi}$  on “When Issued” mode on the calculate date:

$$P_{ex} = P_{com} - TC * F * q * P_{wi}$$

- ii. If there's no price on step i and the underlying asset has a trading price  $P_{wi}$  on “When Issued” mode on the calculate date:

$$P_{ex} = TC * F * P_{wi}$$

- i. If there're no price on “When Issued” mode, the possibility of postponing the calculation date by one day of trading of the underlying asset will be evaluated in order to have the inputs to pricing. In this case, one of following equations are applied depending on the available information (the application of an equation ends the calculation)

- a. New company's price is available

$$P_{ex} = P_{com} - TC * F * q * P_{new}$$

- b. Old company's price is available

$$P_{ex} = TC * F * P_{old}$$

- ii. If it is not possible to postpone the date of the action, B3 will evaluate case-by-case how to calculate, having the possibility of defining  $P_{ex} = P_{com}$ .

#### 1.4 FII or stocks spin-off actions

For the event of spin-off on a new company non-tradable at the computation moment but with a reference market *off-shore*, the ex-benefit price will be computed by reducing the closing price with the benefit by the adjusted *off-shore* price (adjusted by exchange rate and proportionality)

$$P_{ex} = P_{com} - \text{adjusted new company reference price}$$

For the cases where there is no reference market or there is but without enough free float, the price will be computed by the capital reduction

$$P_{ex} = P_{com} * \% \text{ reduction}$$

When there is a FII spin off on a non-tradable FII, for the price of the new FII will be used the one declared by the manager, then

$$P_{ex} = P_{com} - q * FII_{new}$$

where  $q$  is the new FII proportion given to each share holder. If the shareholders have the option of adding or not the spin off, then there is no impact on the price:  $P_{ex} = P_{com}$ .

## 1.5 Several actions on one day

In cases where there is more than one action in the same day, the calculation must be in the same order of actions as declared by the issuing company. The cash actions, percentage bonus and ordinary warrants can be summarized in the following equation (more details on warrants can be checked at next sections)

$$P_{ex} = \frac{P_{com} + w * K - X}{1 + w + B} \quad (1.4)$$

where  $X$  is the total cash action and  $B$  the bonus percentage. The other variables are given at the Notation section. If the equation (1.4) gives a result such that  $P_{ex} \leq K$ , then the warrants is not considered, and the formula used will be

$$P_{ex} = \frac{P_{com} - X}{1 + B}$$

## 2 GENERAL ASSUMPTIONS OF A SUBSCRIPTION

Under the general assumption of maintaining the financial of the shareholder, the ex-price ( $P_{ex}$ ) must satisfy the following relation with price  $P_{com}$ :

$$P_{com} = P_{ex} N + w * E[g(P(S_T), K, T) * D(T)]$$

Where  $w$  is the ratio of the subscription per share, the function  $g$  represents the subscription payoff, and  $P(S_T)$  the value at expiration of the subscribed asset and  $D(T)$  discount factor with time frame  $T$ . Generally, this function  $g$  is the maximum between 0 and the difference between the value of the subscribed asset and the subscription value  $K$ , so:



$$P_{com} = P_{ex} + w * E[máx(P(S_T) - K, 0) * D(T)].$$

In this case, the expected value can be replaced by the premium of a call

$$P_{com} = P_{ex} + w * Call(P(P_{ex}), K, T)$$

Assuming the subscription expires in a maximum of one month, the Call price of a subscription is usually approximated by the payoff, thus

$$P_{com} = P_{ex} + w * max(P(P_{ex}) - K, 0) \quad (2.1)$$

In the following sections, formula (2.1) is applied to different asset type subscription issues. The formulas presented are applicable for events with the following conditions.

- The subscription is advantageous, thus  $P_{com} > K$  and  $P_{ex} > K$ . The bonus is considered to impact the ex-price only if the ordinary subscription is advantageous;
- The period for exercising the subscription is a maximum of one month;
- It is assumed that the subscription will be fully exercised;
- It is assumed that the company will not pay dividends until the expiration date of the subscription or subscription bonus;
- If the subscription bonus has an exercise period, the last date is considered to be the expiration of the bonus;
- In the pricing of subscription assets, the interest rate and the credit spread are considered as deterministic variables;
- For warrants on stock baskets, the stock price on the pricing formulas will be substituted by the synthetic basket price -price computed combining the stock prices with their weights on the basket-.

It should be noted each subscription has its own characteristics that must be analyzed to assess adherence with the calculation models presented in the

section 3 and 4. If the characteristics were not according to the models or variables presented, it could be defined price R\$0,00 for the warrants and the bonus.

### 3 CALCULATING EX-RIGHTS AND SUBSCRIPTION RIGHTS PRICES

#### 3.1 Transferable Share or fund Subscription Issue

The subscription asset is the company's share, thus, in formula (2.1),  $P(P_{ex}) = P_{ex}$ ,

Therefore:

$$P_{ex} = P_{com} - w * \max(P_{ex} - K, 0)$$

Equivalently, assuming, the subscription is advantageous i.e.  $P_{com} > P_{ex} > K$ ,

$$P_{ex} = \frac{P_{com} + w * K}{1 + w} \quad (3.1)$$

Formula (3.1) defines the ex-price on the ex-date. The subscription rights price  $RD$  is given by  $\max(P_{ex} - K, 0)$ .

When the share subscription issue is non-tradable, the value of  $P_{ex}$  and the pricing model depend on information available from the issuing company. If a price for the asset is available, given by the company,  $P(P_{ex})$  will be the informed price. In this case the warrants could be considered not advantageous resulting in  $P_{ex} = P_{com}$  and  $RV = 0$ .

#### 3.2 Subscription Bonus Issue on Warrants

The subscription asset is a warrant, therefore, formula (2.1) is obtained by:

$$P_{com} = P_{ex} + w * \max(\text{Call}(q_a * P_{ex}, \tilde{K}, \tilde{T}) - K, 0). \quad (3.2)$$

Solving equation (3.2) gives the value of  $P_{ex}$ . The solution demands solving a numerical optimization

$$\min_{P_{ex}} \left( P_{com} - P_{ex} - w * \max(\text{Call}(q_a * P_{ex}, \tilde{K}, \tilde{T}) - K, 0) \right)^2$$

The subscription rights price  $RV$  is given by the equation

$$RV = \frac{P_{com} - P_{ex}}{w} \quad (3.3)$$

### 3.3 Share Subscription Issue and Bonus Issue

Under the general assumption, the ex-price must satisfy the equation:

$$P_{com} = P_{ex} + w * \max(P_{ex} - K + q_b * \max(\text{Call}(q_a * P_{ex}, \tilde{K}, \tilde{T}) - K_b, 0), 0) * 1_{P_{com} > K} \quad (3.4)$$

It should be noted the bonus only affects the price when the share subscription is advantageous.

Solving the previous equation it gives the value of  $P_{ex}$ . The solution is given by solving the following numerical optimization problem and the subscription rights price  $RD$  is given by the equation (3.3).

$$\min_{P_{ex}} \left( P_{com} - P_{ex} - w * \max(P_{ex} - K + q_b * \max(\text{Call}(q_a * P_{ex}, \tilde{K}, \tilde{T}) - K_b, 0), 0) * 1_{P_{com} > K} \right)^2 \quad (3.5)$$

In case where the bonus is on stock basket, the *Call* is computed on the basket. The volatility is estimated using the theoretical price historical series of the basket as exposed as A.1. If both stocks give the subscription rights, the ex-prices are given as follows.

$$\begin{aligned}
P_{com}^{ON} &= P_{ex}^{ON} + w^{ON} \\
&\quad * \max(P_{ex}^{ON} - K^{ON} + w^{ON} * q_b^{ON} \\
&\quad * \max(Call(q_a^{ON} * P_{ex}^{ON} + q_a^{PN} * P_{ex}^{PN}, \tilde{K}, \tilde{T}) - w^{ON} * K_b^{ON}, 0), 0) * 1_{P_{com}^{ON} > K^{ON}}
\end{aligned}$$

$$\begin{aligned}
P_{com}^{PN} &= P_{ex}^{PN} + w^{PN} \\
&\quad * \max(P_{ex}^{PN} - K^{PN} + w^{PN} * q_b^{PN} \\
&\quad * \max(Call(q_a^{ON} * P_{ex}^{ON} + q_a^{PN} * P_{ex}^{PN}, \tilde{K}, \tilde{T}) - w^{PN} * K_b^{PN}, 0), 0) * 1_{P_{com}^{PN} > K^{PN}}
\end{aligned}$$

Being  $P_{com}^{ON} = g(P_{ex}^{ON}, P_{ex}^{PN})$  and  $P_{com}^{PN} = h(P_{ex}^{ON}, P_{ex}^{PN})$ , the above equations solution is obtained by the solution of the following optimization.

$$\min_{(P_{com}^{ON}, P_{com}^{PN})} (P_{com}^{ON} - g(P_{ex}^{ON}, P_{ex}^{PN}))^2 + (P_{com}^{PN} - h(P_{ex}^{ON}, P_{ex}^{PN}))^2 \quad (3.5)$$

### 3.4 Bonus Subscription Issue on LFs (Bank Issued Debenture) or Debentures

The subscription asset is an LF (Bank Issued Debenture) or a debenture and its value is denoted by  $PRD$ , the calculation formula is:

$$P_{com} = P_{ex} + w * \max(PRD - K, 0) \quad (3.6)$$

Solving equation (3.6) give the value of  $P_{ex}$ . The subscription rights price  $RD$  is given by the equation (3.3).

The annex of this manual presents some models for pricing debentures.

## 4 CALCULATING THE SUBSCRIPTION RIGHTS AND BONUS PRICE

In this section will be explained the calculation of the rights and the subscription mainly on cases of securities lending. However, the formulas can be used also for other events or products. According to the B3's Clearinghouse Operating Procedures Manual, for securities lending positions, some corporate actions are treated as a cash event and the prices follows the formulas on this section.

The general assumption for pricing the right is that at time  $t$ , the share price  $S_t$  is known, as the ex-event has already affected the share price or because there is no ex-event date.

Following the assumptions and rational of section 2, the general pricing formula for the right  $RV$  is

$$RV = (P(S_t) - K, 0) \quad (4.1)$$

$P(S_t)$  being the value of the subscription asset at the time of calculation.

#### 4.1 Transferable Share or FII Subscription Issues

The value of the right to subscribe to shares after the ex-date is obtained by equation (4.1) where  $P(S_t) = S_t$  and  $t$  the calculation date.

When the share or the FII is non tradable, the  $P(S_t)$  value and the pricing model will depend on the information given by the company. In this case, the results could be  $P_{ex} = P_{com}$  and  $VD = 0$ .

#### 4.2 Subscription Bonus Issue on Warrants

The value for the right to subscription bonus issue on warrants is given by  $P(S_t)$  obtained by the equation:

$$P(t) = \frac{1}{1+w} * Call(S(t) + w * P(t), \tilde{K}, \tilde{T}) \quad (4.2)$$

Solved through a numerical optimization problem.

#### 4.3 Share Subscription Issues and Bonus Issues

The value of the right for a subscription in shares with subscription bonuses is given by the solution of the following equation

$$P(t) = \frac{1}{1+w*q_a} Call(S(t) + w * P(t) + w * q_b * (Z - K_b), K, \tilde{T}) \quad (4.3)$$

Where, for simplicity,  $Z = Call(q_a * S(t), \tilde{K}, \tilde{T})$ .

If just price for the bonus is needed and the subscription issue was traded on the day of the calculus with price  $VD$ , it can be assumed that  $VD = q_b * máx(bônus - K_b, 0) + máx(S_t - K, 0)$  and then the bonus will be computed as follows

$$bônus = \begin{cases} \frac{VD - máx(S_t - K, 0)}{q_b} + K_b, & \text{se } VD - máx(S_t - K, 0) > 0 \\ 0, & \text{caso contrário} \end{cases} \quad (4.4)$$

For the cases when the warrant gives more than one bonus, the result of equation (4.4) will be used for get the sum of all bonuses prices. In case of the warrant have not traded on the computation day, the bonus price will be computed as described in section 4.5.

#### 4.4 Subscription Bonus Issue on LFs (Bank Issued Debentures) or Debentures

The reference price for a debenture subscription right is calculated using equation (4.1) where  $P(t) = PRD$ , where  $PRD$  is the reference price of the debenture.

#### 4.5 Bonus issue on tradable asset or FII

The corporate events that give bonus issue on tradeable asset or FII may need price in some instances such as:

- 1- When the bonus is tradeable, it will be defined the price on the first day for open the market;
- 2- When a security lending positions needs bonus price B3's Clearinghouse Operating Procedures Manual.

For any corporate action which bonus price is needed, it will be the closing auction price. In case there is no negotiation during the auction, the price will

be computed as a European call option  $Call(q_a * S(t), K, T)$  using the Black & Scholes model shown as section A.5, the volatility computed as described on section A.1. The expiration day  $T$  will be the longest exercise date defined by the company.

## 5 CALCULATING OTHER BENEFITS

### 5.1 Acquisition with bonus actions

When the two companies are listed on B3, the bonus price is calculated as follows.

$$bonus = P_{mother}(t) - P_{child}(t)$$

Where

$P_{mother}$  = acquiring company price on date  $t$

$P_{child}$  = acquired company price on date  $t$

### 5.2 Bankruptcy or issuer's out-of-court liquidation

According to the B3's Clearinghouse Operating Procedures Manual, this section provides the steps to determine the price that will be used to settle open positions of derivatives and loans in case of bankruptcy of a company listed on B3. Following the B3's Trading Procedures Manual, B3 can establish a specific auction market to determine the reference price.

The auction price  $P_{auction}(t)$  will be defined as follows:

- i. In case where the number of stocks traded is bigger than the validation parameter:

$$P_{auction}(t) = P_{trade}(t)$$

where  $P_{trade}(t)$  is the price of the trades during the auction.

- ii. In case where it's not possible to define the price as step i will be considered the mean price of the best buy offers present at the end of the auction and all the trades that totalize the validation parameter:

$$P_{\text{auction}}(t) = \frac{q_{\text{trade}} * P_{\text{trade}}}{Q_{\text{min}}} + \frac{\sum_n q_n * P_n}{Q_{\text{min}}}$$

where:

$P_n$  is the price of the buy offers from the level n of the book;

$Q_{\text{min}}$  validation parameter;

$q_{\text{trade}}$  quantity of stocks traded during the auction;

$P_{\text{trade}}$  price of the trades during the auction;

$q_1 = \min\{Q_1, Q_{\text{min}} - q_{\text{trade}}\}$ ;

$q_n = \min\{Q_n, Q_{\text{min}} - q_{\text{trade}} - \sum_{j=1}^{n-1} q_j\}$  when  $n > 1$ ;

$Q_n$  is the quantity of stocks of the level n of the book.

iii. In case where it's not possible to define the price as step ii:

$$P_{\text{auction}}(t) = R\$ 0,00$$

The validation parameter will be expressed as quantity of stocks.

B3 can define the maximum or minimum price to register the offers.

### 5.3 Options on non-tradable stock price

When the underlying of an option is no more listed for trading at B3, B3 can decided to close out the positions on that option through a cash liquidation. For those cases the option price  $P$  is given by it payoff:

- Call option,  $P = \text{Maximum}(PA - PE, 0)$
- Put option,  $P = \text{Maximum}(PE - PA, 0)$

where

PA= underlying price for this cash treatment. For bankruptcy cases the price is defined by the methodology of section 5.2. For buyback cases, the price will follow the rules defined by the company on the notice of the event. If there will not be a special auction for the event, the price will be the last one traded.

PE= option strike.



#### **5.4 Corporate action delivering non tradable stock**

When the corporate action gives a non tradable stock, the price used will be the one declared in the event notice. If there is underlying market for the stock, the price will be the one of the underlying market adjusted by the stock characteristics.

## A SUPPLEMENTARY FORMULAS

### A.1 Volatility

When it is necessary to use the share volatility for time-frame  $T$ , denoted by  $\sigma_T$ , it is estimated by one of the following methods

- i. When the event has an exercise price, the volatility will follow the methodology described at the Options pricing Manual.
- ii. Otherwise, it will be computed by using formula (A.1), being an estimate of the temporal volatility from a GARCH model (1.1) with normal residuals.

$$\sigma_T = \sqrt{252 V_T} \quad (\text{A.1})$$

for

$$V_T = V_L + \frac{1 - \exp(-\theta T \cdot 252)}{\theta T \cdot 252} (\hat{\sigma}^2(t + 1) - V_L)$$

Where  $\theta = \ln \ln \left( \frac{1}{\alpha + \beta} \right)$  and the coefficients  $\omega, \alpha, \beta$  estimated on the return series  $x(t)$  of the action using the maximum likelihood technique, according to equation (A.2) and (A.3)

$$\hat{\sigma}^2(t) = \omega + \alpha x^2(t - 1) + \beta \hat{\sigma}^2(t - 1) \quad (\text{A.2})$$

$$x(t) = \sqrt{\hat{\sigma}^2(t)} z_t \quad (\text{A.3})$$

where  $z_t$  follows a standard Gaussian distribution and

$$V_L = \frac{\omega}{1 - \alpha - \beta} \quad (\text{A.4})$$

It should be noted that for shares with low liquidity, the number of null returns may make the estimation of volatility by the GARCH model unfeasible. In such cases, the historical volatility estimated in a period equivalent to the time between the calculation date and the exercise date

will be used or the volatility of a similar share will be used in the classification by segment.

## A.2 Value of an LF (Bank Issued Debentures) or Non-Convertible Debenture

The reference price for an LF or debenture will be calculated according to equation (A.5), considering variables from the market's close on the calculation day, provided that the issuing characteristics adhere to the following model:

$$PRD = \sum_{i=1}^N \frac{Interest_i \times VNU_i + PA_i \times VNU}{(1+r_i)^{pr_i} \times (1+spr_{cred})^{pr_i}} \quad (A.5)$$

where:

*PRD*: reference price of the debenture or LF.

*i* : indicator referring to the interest payment and amortization dates.

*N*: number in the payments order.

*Interest<sub>i</sub>*: remunerative interest factor paid on date *i*, calculated according to equation (A.6).

*r<sub>i</sub>*: fixed rate for the time frame corresponding to the payment date *i*, calculated using exponential interpolation of the settlement prices from the One-day Interbank Deposit Futures Contract (DI1).

*VNU*: issue par value.

*VNU<sub>i</sub>*: Par value on date *i*, which corresponds to the par value of the issue minus the amortized percentage, according to the amortization schedule given in the LF documentation.

*PA<sub>i</sub>*: percentage amortized on date *i*, according to the amortization schedule given in the LF documentation.

$pr_i$ : time frame in years corresponding to date  $i$ , calculated from the business days in the period.

$spr_{cred}$ : credit spread, on an annual basis, referring to issues with similar credit characteristics and terms.

The remuneration interest factor is determined by equations (A.6) and (A.7)

$$Interest_i = (1 + p \times CDI_{Proj})^n - 1 \quad (A.6)$$

$$CDI_{Proj} = (1 + r_i)^{\frac{1}{252}} - 1 \quad (A.7)$$

where:

$n$ : number of business days in the interest period.

$p$ : percentage of the CDI rate to be paid.

Equation (A.6) is used as a proxy for the equation

$$Interest_i = \prod_{s=0}^{i-1} \left( \left[ (1 + r(s, s+1))^{1/252} - 1 \right] * p + 1 \right) - 1$$

for  $r(s, s+1)$ : the spread between time frames and the next day implied in the settlement prices of the One-day Interbank Deposit Futures Contract (DI1).

### A.3 Value of Debentures Convertible into Shares

The debenture's reference price is calculated using a binomial tree, which calculates the conversion into shares every day during the conversion period, provided the issue characteristics are consistent with the model herein proposed. The conversion assessment payoff is differentiated by (i) expiration date, (ii) dates included in the conversion period and (iii) other dates. The reference price of the debenture is denoted by  $PRD$  and the price on date  $T$  in scenario  $j$  by  $PRD(T, j)$ .

(i) Expiration date, by the assumption of mandatory conversion at expiration,

$$PRD(T, j) = S_c(T, j)Q_c(T, j) \quad (A.8)$$

where

$j$ : index that represents the share price scenario at each instant of valuation.

$T$ : is the maturity date of the debenture.

$S_c(T, j)$ : share price at time  $T$  and in the price scenario  $j$ , calculated from the binomial tree.

$Q_c(T, j)$ : number of shares received from the conversion at time  $T$  and in the price scenario  $j$ , according to parameters established in the debenture documentation.

(ii) Dates included in the conversion period.

From the values  $PRD(T, j)$  of each scenario  $j$ , you can work backwards through each node to the first, calculating the expected values of the decisions of each time point  $i$ , discounted by the rate of interest and credit spread.

$$PRD(i, j) = \left[ \frac{p PRD(i + 1, j + 1) + (1-p)PRD(i + 1, j)}{(1 + r(T_{i-1}, T_i))^{\frac{1}{252}}(1 + spr_{cred})^{\frac{1}{252}}}; S_c(i, j)Q_c(T, j) \right] \quad (A.9)$$

where

$i$ : index that represents the date or time of valuation, considers daily steps up to the maturity date of the debenture.

$p$ : probability associated with the reference price  $PRD(i + 1, j + 1)$ , calculated according to equation (A.12).

$r(T_{i-1}, T_i)$ : spread between  $T_{i-1}$  and  $T_i$ , calculated through the exponential interpolation of the settlement prices of the DI1 Futures Contracts.

$spr_{cred}$ : credit spread. The spread implicit in the unit price of the debenture can be used.

(iii) Other dates

For other dates before and after the conversion date, the reference price of the convertible debenture,  $PRD(i, j)$  is calculated from the equation

$$PRD(i, j) = \frac{p PRD(i+1, j+1) + (1-p) PRD(i+1, j)}{(1 + r(T_{i-1}, T_i))^{\frac{1}{252}} (1 + spr_{cred})^{\frac{1}{252}}} \quad (\text{A.10})$$

Shocks applied to asset prices  $u$  and  $d$  are given by

$$u = \exp(\sigma_T \sqrt{\delta}) \quad \text{and} \quad d = \frac{1}{u} \quad (\text{A.11})$$

where

$\delta$ : time interval for evaluating the conversion option, considered 1 business day.

$\sigma_T$ : volatility of each share, calculated by equation (A.1)

The probability  $p$ , used in expression (A.9) and (A.10), associated with each node  $(i, j)$  is calculated by the following equation

$$p = \frac{\exp(r(T_{i-1}, T_i)\delta) - d}{ud} \quad (\text{A.12})$$

#### A.4 Value of Perpetual Debentures Convertible into Shares

The reference price for perpetual debentures convertible into shares is denoted by  $PNR$  and the par value of the debentures by  $VNU$ .  $PNR$  is calculated considering the following assumptions:

- the perpetual debentures can be converted into shares at any time.

- to simulate the perpetuity effect, a maximum time frame is defined in years  $T_M$ , typically being 20 (twenty) years.
- the remuneration of the debenture on date  $t$  is denoted by  $Rem_t$ , this remuneration can be linked to company profit  $R_i$ ,  $Rem_t = VNU + \sum_{i=t}^{T_M} R_i$  or the remuneration can be a payment of interest with amortization

$$Rem_t = \sum_{i=t}^N \frac{(Interest_i \times VNU_i + PA_i \times VNU)}{\left((1 + r(t, T_i))(1 + spr_{cred})\right)^{(T_i - t)}}$$

The share price is generated using a random walk of daily septs given by the following expression

$$S_t = S_{t-1} \cdot \left[ \left( r - \frac{1}{2} \sigma^2 \right) \frac{1}{252} + \sqrt{\frac{1}{252}} \sigma \epsilon \right] ; \epsilon \sim N(0,1)$$

where

$S_0$ : the price on the share calculation date.

$r$ : a fixed interest rate (continuous rate) for the time frame  $T_M$ .

$N(0,1)$ : standard normal distribution.

$\sigma^2 = 252 V_L$  para  $V_L$  the volatility obtained according to equation (A.4).

For each simulated date, the conversion of the debenture into shares occurs when  $\gamma S_t \geq Rem_t$ , otherwise, the debenture is not converted.  $\gamma$  is the conversion factor per share.

The  $PNR$  is the average of the present value of all scenarios simulated. In scenarios in which there is conversion, the present value of the share is considered in the period in which the conversion occurs. In the others, the present value of the debenture is considered within the maximum time frame

established. The calculation of the present value considers, in the discount factor, the fixed interest rate and the credit spread of the issuing company.

### A.5 Black & Scholes option model

The European option pricing model used at this manual is Black&Scholes model given by

$$Call = S \times N(d_1) - K \times e^{(-r_n T_n)} \times N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma_T^2}{2}\right)T}{\sigma_T \sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma_T^2}{2}\right)T}{\sigma_T \sqrt{T}}$$

$r$ : interest rate for time  $T$ .

$N$ : standard normal distribution.

$K$ : options strike.

$S$ : underlying price.



## Change Log

Version	Item modified	Modification	Date
1	Original version	-	08/03/2020
2	Title	Change	12/22/2022
	Section 1	Addition	
	Section 2	Paragraph 1 with simplification of the notation	
	Section 3.3	Formula changes	
	Section 4	Changes on formula (4.3) and (4.4)	
	Section 5	Addition	
3	Section 5.2	Addition	01/10/2023
4	Section 5.2	Addition of trades at item ii	02/23/2023
5	Section 5.2	Adjust on the text and formula at item ii	09/03/2023
8	Section 3.1	Inclusion final paragraph	06/09/2023
	Section 4.1	Addition final paragraph	
	Section 4.3	Change on bonus calculation	
	Section 5.3	Addition	
	Section A.1	Addition methodology i.	
	Section A.5	Addition	
9	Section 5.3	Correction including the closing auction price	06/29/2023
10	General	Text and formulas typing adjustments and change of section 5.3 to 4.5	12/15/2023
	Section 4.5	Addition, at previous version it was section 5.3	
	Section 1.4, 1.5, 5.3 and 5.4	Addition	

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